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Two-particle interference in standard and Bohmian quantum mechanics

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Abstract

The compatibility of standard and Bohmian quantum mechanics has recently been challenged in the context of two-particle interference, both from a theoretical and an experimental point of view. We analyse different setups proposed and derive corresponding exact forms for Bohmian equations of motion. The equations are then solved numerically, and shown to reproduce standard quantum-mechanical results.

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1. Introduction

Just about 50 years ago, Bohm proposed a new interpretation of quantum mechanics [1]. Although standard quantum mechanics (SQM) is statistical and nondeterministic, Bohmian quantum mechanics (BQM) is fully causal. In BQM the wave–particle duality is resolved. A quantum object is a particle with well-determined, although not accurately known, position and momentum. The quantum object's wave characteristics are embodied in a quantum potential that acts on the particle and is related to the wavefunction. In a system of n particles with total wavefunction $\Psi(\mathbf{r}_1, \dots, \mathbf{r}_n, t)$, the velocity of particle i is taken to be [1–3]

$$\dot{\mathbf{r}}_i = \frac{\hbar}{m_i} \operatorname{Im} \left[\frac{\nabla_i \Psi(\mathbf{r}_1, \dots, \mathbf{r}_n, t)}{\Psi(\mathbf{r}_1, \dots, \mathbf{r}_n, t)} \right]. \quad (1)$$

In BQM, the probability density of finding particles $1, \dots, n$ at points $\mathbf{r}_1, \dots, \mathbf{r}_n$ is given, as in SQM, by the absolute square of the total wavefunction. The BQM probability is subjective, that is, it expresses our ignorance of the particles' true positions. The continuity equation satisfied by the probability density implies that the latter is consistent with Bohmian trajectories. This means that if Bohmian particles are distributed according to the absolute square of the wavefunction at time t_0 , then their trajectories will transform their probability distribution precisely as SQM predicts. The probability of finding a given particle at a given

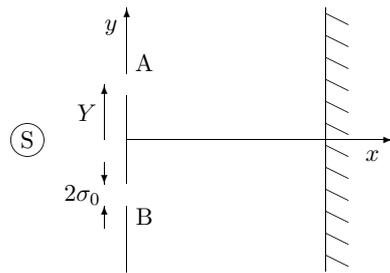


Figure 1. Double-slit setup. Correlated pairs of particles emitted from source S go through slits A and B and are detected on a screen.

position is the same in both interpretations. Thus, statistical predictions of SQM and BQM should be indistinguishable.

Nevertheless, Ghose [4] as well as Golshani and Akhavan [5–7] recently proposed experimental setups that allegedly lead to different predictions for SQM and BQM. An experiment along these lines has been carried out by Brida *et al* [8], and the results were interpreted as confirming SQM and contradicting BQM. All proposed experiments are based on two-particle interference, and make use of symmetrical arrangements of either two or four slits. The disagreement between SQM and BQM is seen in the fact that SQM statistically allows some pairs of particles to reach detectors asymmetrically, while Bohmian trajectories are claimed to forbid this. These conclusions have been disputed by Struyve *et al* [9] and by one of us [10]. The objection is that references [4–7] make use of unwarranted hypotheses on the initial distribution of the particles' positions.

In this paper we first examine the double-slit and two-double-slit setups proposed by Golshani and Akhavan, showing that in all important aspects the latter in fact reduces to the former. Next we obtain the equations of motion exactly and point out general properties of the velocities. We then compute Bohmian trajectories numerically. The trajectories are used to show explicitly how the agreement between SQM and BQM comes about.

2. Experimental setups

2.1. Double-slit setup

Two-particle interference differs from one-particle interference in that the interference pattern does not show up in the individual detection of a particle on a screen but in the joint detection of a pair of particles. That is, the interference pattern is a property of configuration space.

The double-slit setup proposed in [4, 6] is shown in figure 1. A pair of identical particles emitted by a source S impinges on a double-slit interferometer. Just behind the slits the wavefunction is assumed to be well described by plane waves with zero total momentum in the y -direction and identical momenta in the x -direction. Thus each particle goes through a different slit. Since both particles are identical, the total wavefunction is symmetrical (bosons) or antisymmetrical (fermions) under particle permutation. The edges of the slits are assumed smooth enough to avoid treating diffraction of wave packets as these go through the apertures [2, p 177]. Each slit is taken to generate a Gaussian wave form in the y -direction, while leaving the wave form unaffected in the x -direction. Explicitly, let particle i emerge from slit A at $t = 0$. Its wavefunction is then written as

$$\psi_A(x_i, y_i, t = 0) = (2\pi\sigma_0^2)^{-1/4} \exp \left\{ -\frac{(y_i - Y)^2}{4\sigma_0^2} + i[k_x x_i + k_y (y_i - Y)] \right\}. \quad (2)$$

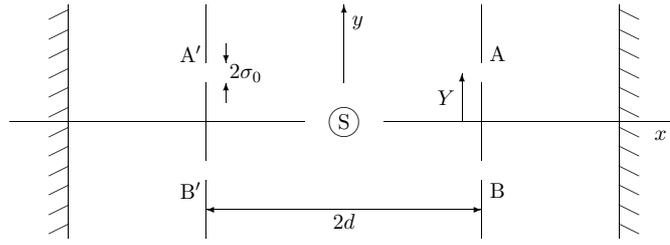


Figure 2. Two-double-slit setup. Correlated pairs of particles emitted from source S go through slits A, B' or B, A' and are detected on screens.

We should point out that once we settle on equation (2) as the one-particle wavefunction, no information on the particle's state prior to $t = 0$ is relevant that is not contained in (2). On the other hand, assuming free propagation with time we get for $t > 0$

$$\psi_A(x_i, y_i, t) = (2\pi\sigma_t^2)^{-1/4} \exp\{-(y_i - Y - \hbar k_y t/m)^2/(4\sigma_0\sigma_t)\} \times \exp\{i[k_x x_i + k_y(y_i - Y - \hbar k_y t/(2m)) - \hbar k_x^2 t/(2m)]\} \quad (3)$$

where

$$\sigma_t = \sigma_0 \left(1 + \frac{i\hbar t}{2m\sigma_0^2}\right). \quad (4)$$

The two-particle total wavefunction is taken as

$$\Psi = N[\psi_A(x_1, y_1, t)\psi_B(x_2, y_2, t) \pm \psi_A(x_2, y_2, t)\psi_B(x_1, y_1, t)] \quad (5)$$

where N is a normalization constant and

$$\psi_B(x, y, t) = \psi_A(x, -y, t). \quad (6)$$

In (5) the plus sign refers to bosons and the minus sign to fermions. Note that the wavefunction Ψ is symmetric (or antisymmetric) with respect to a reflection in the $y = 0$ plane.

2.2. Two-double-slit setup

A two-double-slit setup was proposed in [5] and is shown in figure 2. A pair of correlated particles leaves source S in a state of zero total momentum. The particles therefore either go through slits A and B' or through slits B and A'. Accordingly, up to a multiplicative factor the total wavefunction was taken in [5] as¹

$$\Psi = \psi_A(x_1, y_1, t)\psi_{B'}(x_2, y_2, t) \pm \psi_A(x_2, y_2, t)\psi_{B'}(x_1, y_1, t) + \psi_{A'}(x_2, y_2, t)\psi_B(x_1, y_1, t) \pm \psi_{A'}(x_1, y_1, t)\psi_B(x_2, y_2, t) \quad (7)$$

with

$$\psi_{A'}(x, y, t) = \psi_A(-x, y, t) \quad (8)$$

$$\psi_{B'}(x, y, t) = \psi_B(-x, y, t) = \psi_A(-x, -y, t). \quad (9)$$

¹ This wavefunction is symmetric with respect to a reflection in the $y = 0$ plane, for both bosons and fermions. As with (5), we could easily make it symmetric for bosons and antisymmetric for fermions, by permuting the sign factors of the third and fourth terms. The following argument would then be carried out just as easily, except that the final correspondence between equations (11) and (12) and equation (5) would involve in the latter both the plus and minus signs.

It is not difficult to check that if the plus sign is picked in (7), the global wavefunction can be written as

$$\Psi = \cos[k_x(x_1 - x_2)]\Phi(y_1, y_2, t). \quad (10)$$

If the minus sign is picked instead, the cosine is simply replaced by a sine. Hence the x_1, x_2 dependence factors out and it is given by a real function. From equation (1), we immediately conclude that $\dot{x}_1 = 0 = \dot{x}_2$. That is, the Bohmian particles do not move in the x -direction.

The reason for this is that with the one-particle wavefunctions written as in (3), (6), (8) and (9), equation (7) does not adequately represent the interference situation depicted in figure 2. Indeed, all one-particle wavefunctions are infinitely extended in both the positive and negative x -directions. Clearly, however, equations (3) and (6) for ψ_A and ψ_B are intended to hold only where $x > d$, while (8) and (9) for $\psi_{A'}$ and $\psi_{B'}$ hold only where $x < -d$. The total wavefunction should therefore be written as

$$\Psi = N'[\psi_A(x_1, y_1, t)\psi_{B'}(x_2, y_2, t) + \psi_{A'}(x_2, y_2, t)\psi_B(x_1, y_1, t)] \quad (11)$$

if $x_1 > d$ and $x_2 < -d$, and as

$$\Psi = \pm N'[\psi_A(x_2, y_2, t)\psi_{B'}(x_1, y_1, t) + \psi_{A'}(x_1, y_1, t)\psi_B(x_2, y_2, t)] \quad (12)$$

if $x_1 < -d$ and $x_2 > d$. The two expressions of the total wavefunction do not overlap in configuration space, whence Bohmian trajectories associated with one are completely independent of the other. As they should, (11) and (12) transform into each other (with a plus sign for bosons and a minus sign for fermions) under particle permutation.

It is easy to check that, up to a constant multiplicative factor, equation (11) can be obtained from (5) (with the plus sign) through the substitution $x_2 \rightarrow -x_2$. From (1) we see at once that all components of velocities are the same with both wavefunctions, except that \dot{x}_2 is transformed into $-\dot{x}_2$. We conclude that Bohmian trajectories in the two situations are in one-to-one correspondence, with x_2 being reflected in the yz -plane.

Exactly the same argument shows that Bohmian trajectories computed with (12) and (5) are also in one-to-one correspondence, with x_1 now being reflected in the yz -plane.

3. SQM and BQM predictions

In the remainder of this paper we shall discuss the double-slit setup only, since results pertaining to the two-double-slit setup can be obtained by straightforward transformation.

Let one pair of particles leave the slits at $t = 0$ and arrive at detectors at time t (both particles have x -momentum equal to $\hbar k_x$). In SQM, the probability of finding the particles on detectors at points y_1 and y_2 is given by

$$P(y_1, y_2, t) = |\Psi(y_1, y_2, t)|^2. \quad (13)$$

Suppose first that t is such that $|\sigma_t| \approx \sigma_0$. Then the spreading of the wave packets is not very important. Therefore, the probability of finding both particles on the same side of the x -axis is very low. The particles will be detected on both sides of the x -axis, rather symmetrically if σ_0 is much smaller than Y (see figure 1). Moreover, as one-particle wavefunctions overlap very little, interference effects will be negligible.

Assume now that $|\sigma_t| \gg \sigma_0$. This can be obtained either by taking detectors further to the right, or by reducing the value of k_x . Then wavefunction overlap becomes important and interference effects begin to show up. Detection becomes more and more asymmetrical, and both members of a pair can even be detected on the same side of the x -axis.

In BQM, particle velocities can be computed from equation (1) and wavefunction (5). The calculation somewhat simplifies if $k_y = 0$, which we henceforth assume. Substituting (3)

and (6) into (5) and discarding multiplicative factors that do not depend on r_1 and r_2 , we find that

$$\Psi \sim \exp\{i[k_x(x_1 + x_2)]\} \left\{ \exp\left[-\frac{y_1^2 + y_2^2 + 2Y(y_2 - y_1)}{4\sigma_0\sigma_t}\right] \pm \exp\left[-\frac{y_1^2 + y_2^2 - 2Y(y_2 - y_1)}{4\sigma_0\sigma_t}\right] \right\}. \quad (14)$$

Making use of (1) we immediately see that

$$\dot{x}_1 = \frac{\hbar k_x}{m} = \dot{x}_2. \quad (15)$$

The y -components are trickier. Discarding now the x_1 and x_2 dependence of the wavefunction and making use of (4), we can write

$$\Psi \sim \exp(-f)\{\exp(-g) \pm \exp(g)\} \quad (16)$$

where

$$f = \left\{ 4\sigma_0^2 \left[1 + \left(\frac{\hbar t}{2m\sigma_0^2} \right)^2 \right] \right\}^{-1} \left[1 - \frac{i\hbar t}{2m\sigma_0^2} \right] (y_1^2 + y_2^2) \quad (17)$$

$$g = \left\{ 4\sigma_0^2 \left[1 + \left(\frac{\hbar t}{2m\sigma_0^2} \right)^2 \right] \right\}^{-1} \left[1 - \frac{i\hbar t}{2m\sigma_0^2} \right] 2Y(y_2 - y_1). \quad (18)$$

From (1) we get

$$\dot{y}_1 = \frac{\hbar}{m} \text{Im} \left\{ -\frac{\partial f}{\partial y_1} + \left(\frac{\partial g}{\partial y_1} \right) \frac{-\exp(-g) \pm \exp(g)}{\exp(-g) \pm \exp(g)} \right\}. \quad (19)$$

Straightforward manipulations and use of trigonometric identities finally yield

$$\dot{y}_1 = -\frac{2\hbar Y m \sigma_0^2 \sin(\hbar t \alpha) \pm Y \hbar^2 t \sinh(2m\sigma_0^2 \alpha)}{(\hbar^2 t^2 + 4m^2 \sigma_0^4) [\cos(\hbar t \alpha) \pm \cosh(2m\sigma_0^2 \alpha)]} + \frac{\hbar^2 t y_1}{\hbar^2 t^2 + 4m^2 \sigma_0^4} \quad (20)$$

where the upper sign is for bosons, the lower sign for fermions and

$$\alpha = \frac{2Y m (y_1 - y_2)}{\hbar^2 t^2 + 4m^2 \sigma_0^4}. \quad (21)$$

Similarly,

$$\dot{y}_2 = \frac{2\hbar Y m \sigma_0^2 \sin(\hbar t \alpha) \pm Y \hbar^2 t \sinh(2m\sigma_0^2 \alpha)}{(\hbar^2 t^2 + 4m^2 \sigma_0^4) [\cos(\hbar t \alpha) \pm \cosh(2m\sigma_0^2 \alpha)]} + \frac{\hbar^2 t y_2}{\hbar^2 t^2 + 4m^2 \sigma_0^4}. \quad (22)$$

When t is large enough, \dot{y}_1 and \dot{y}_2 are dominated by the last term in (20) and (22). The behaviour of fermions and bosons therefore coincide.

It is easy to check that velocity components satisfy the following relations:

$$\dot{y}_i(x_1, y_1; x_2, y_2; t) = -\dot{y}_i(x_1, -y_1; x_2, -y_2; t) \quad (i = 1, 2) \quad (23)$$

$$\dot{y}_1(x_1, y_1; x_2, y_2; t) = -\dot{y}_2(x_1, -y_2; x_2, -y_1; t). \quad (24)$$

Furthermore, the y -component of the centre-of-mass coordinate satisfies

$$\dot{y}(t) \equiv \frac{\dot{y}_1(t) + \dot{y}_2(t)}{2} = \frac{\hbar^2 t y}{\hbar^2 t^2 + 4m^2 \sigma_0^4}. \quad (25)$$

This is readily integrated as [6]

$$y(t) = y(0) \left\{ 1 + \left(\frac{\hbar t}{2m\sigma_0^2} \right)^2 \right\}^{1/2} = y(0) \frac{|\sigma_t|}{\sigma_0}. \quad (26)$$

If two particles emerge from the slits at heights symmetrical with respect to the x -axis, then $y(0) = 0$. Equation (26) implies that $y(t) = 0$ for all t , and the particles will necessarily be detected symmetrically. Likewise if $y(0)$ is very small, specifically, much smaller than σ_0 , $y(t)$ should remain small enough so that the two particles will be detected almost symmetrically.

From such observations, Ghose [4] and Golshani and Akhavan [7] argued that BQM could make experimental predictions beyond what SQM allows. One would only have to prepare a number of pairs each with $|y(0)| \ll \sigma_0$, and detect them at time t . We have seen that, if $|\sigma_t| \gg \sigma_0$, SQM predicts highly asymmetrical detection. Yet with $y(0)$ small enough, BQM would predict highly symmetrical detection.

The flaw in the argument was pointed out in [9, 10], where it was shown that such a selection of $y(0)$ values is incompatible with Bohm's assumptions. It is instructive to make the argument fully quantitative. In Bohm's theory, the probability distribution of particle positions is given by the absolute square of the wavefunction. Making use of (5), (2) and (6), we see that at $t = 0$ the distribution of y -coordinates is given by ($k_y = 0$)

$$P(y_1, y_2, 0) = |N|^2 (2\pi\sigma_0^2)^{-1} \{F + G \pm 2H\} \quad (27)$$

where

$$F = \exp \left\{ -\frac{(y_1 - Y)^2 + (y_2 + Y)^2}{2\sigma_0^2} \right\} \quad (28)$$

$$G = \exp \left\{ -\frac{(y_2 - Y)^2 + (y_1 + Y)^2}{2\sigma_0^2} \right\} \quad (29)$$

$$H = \exp \left\{ -\frac{y_1^2 + y_2^2 + 2Y^2}{2\sigma_0^2} \right\}. \quad (30)$$

Straightforward integration shows that the probability is normalized to one if

$$|N|^2 = \frac{1}{2} \left\{ 1 \pm \exp \left(-\frac{Y^2}{\sigma_0^2} \right) \right\}^{-1}. \quad (31)$$

It is easy to check that $\langle y_1 + y_2 \rangle = 0$. The standard deviation of $y(0)$ then follows from an evaluation of $\langle (y_1 + y_2)^2 \rangle$. All integrations are elementary, and we obtain

$$\Delta y(0) = \frac{1}{2} \sqrt{\langle (y_1 + y_2)^2 \rangle} = \frac{\sigma_0}{\sqrt{2}}. \quad (32)$$

Hence it is not possible, in Bohm's theory, to select initial positions so that $|y(0)| \ll \sigma_0$. Any such selection scheme amounts either to picking a different initial wavefunction, or to making assumptions on the distribution of true particle positions different from Bohm's original ones.

4. Trajectories

To investigate Bohmian trajectories of pairs of particles governed by wavefunction (5), we have computed them numerically. A fourth- and fifth-order Runge–Kutta algorithm was implemented to solve differential equations (20) and (22), both in Mathematica and through a special purpose program we wrote in C. Results displayed all use m equal to the mass of the electron, $\sigma_0 = 10^{-6}$ m, $Y = 5\sigma_0$ and $k_y = 0$. The distance between slits and detectors is set at 0.2 m. From (15) we see that the time needed for a particle to reach the detector is inversely proportional to k_x .

Figure 3 shows a number of pairs of trajectories for two different values of k_x , with the result that $|\sigma_t| = 1.16\sigma_0$ in (a) and $|\sigma_t| = 5.88\sigma_0$ in (b). For each pair, y -coordinates were

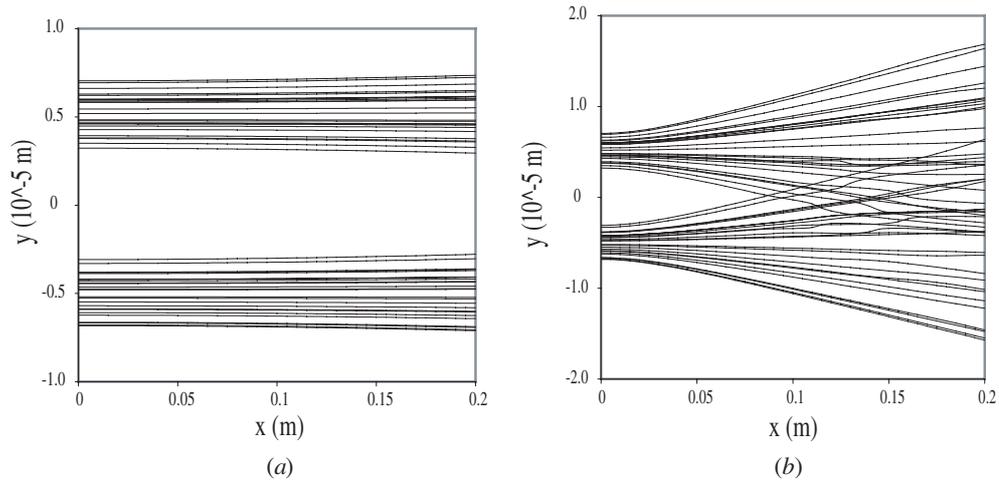


Figure 3. Twenty-five pairs of trajectories for (a) $\hbar k_x/m = 2 \times 10^7 \text{ m s}^{-1}$ and (b) $\hbar k_x/m = 2 \times 10^6 \text{ m s}^{-1}$.

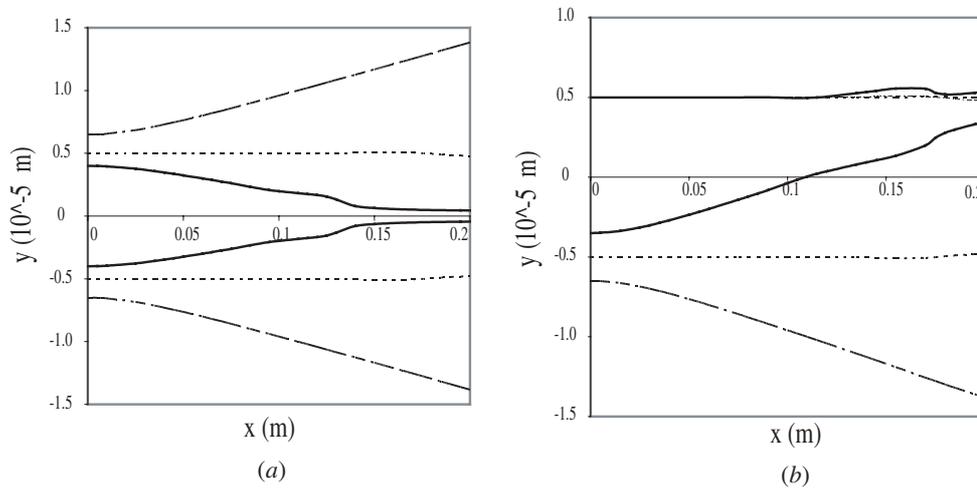


Figure 4. Three pairs of trajectories with (a) symmetrical and (b) asymmetrical initial conditions.

picked randomly according to Gaussian distributions with standard deviation σ_0 , one centred about Y and the other centred about $-Y$. In (a) the Bohmian trajectories are almost horizontal straight lines, which bears out the fact that wave packets have spread very little. In (b), on the other hand, Bohmian trajectories spread and a number of pairs display considerable interference. Some particles coming out of the upper slit end up below the x -axis, and some coming out of the lower slit end up above.

Figure 4 shows a much smaller number of pairs, this time labelled independently. In (a) the initial y -coordinates of both particles are picked symmetrically, that is, $y(0) = 0$. Trajectories remain symmetrical, as equation (26) predicts. In (b), on the other hand, the initial y -coordinates of the upper particles are set at $y = Y$, while the y -coordinates of the lower particles are picked as $y = -Y + 1.5\sigma_0$ (solid line), $y = -Y$ (dotted line) and $y = -Y - 1.5\sigma_0$ (dot-dashed line), respectively. The particle leaving from $y = -Y + 1.5\sigma_0$ clearly ends up above the x -axis, and it gets arbitrarily far from the axis if allowed to go on.

To sum up, the statistical distribution of Bohmian trajectories is fully consistent with the predictions of standard quantum mechanics. Moreover, pairs of Bohmian particles can be detected on the same side of the setup's symmetry plane, and arbitrarily far from the plane.

In the experiment reported in [8], pairs of photons generated by parametric down-conversion were allowed to go through a double-slit setup of the type shown in figure 1. A number of pairs were detected on the same side of the symmetry plane. On the basis of symmetric photon trajectories obtained in [11], and following the analysis of electron like trajectories carried out in [4, 7], this result was interpreted as confirming SQM against BQM.

It is clear that our analysis leads to a different picture. We have shown that nothing prevents Bohmian particles from being detected on the same side of the symmetry plane. Hence, under the assumption that electron trajectories are in that respect relevant to photons, the results of [8] should be interpreted as confirming both SQM and BQM.

5. Conclusion

In this paper, double-slit and two-double-slit setups for two-particle interference were analysed and shown to be essentially equivalent. Bohmian equations of motion for pairs of bosons or fermions were then obtained and numerically integrated. The statistical distribution of Bohmian trajectories turns out to reproduce standard quantum-mechanical results. Members of a given pair are not restricted to remain on different sides of a symmetry plane. Relevant experimental results are therefore consistent with both SQM and BQM.

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